Multi-Axial Fatigue Life Prediction under Non-Proportional Variable-Amplitude Loading Conditions

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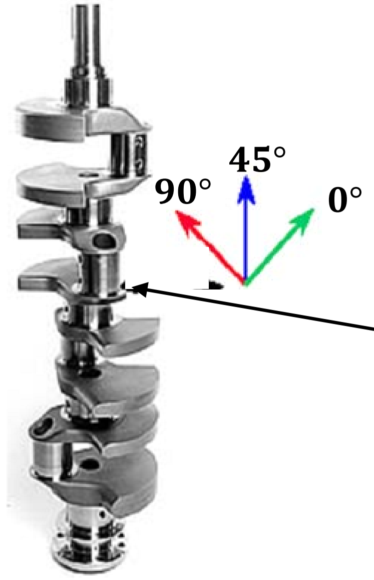
Outline

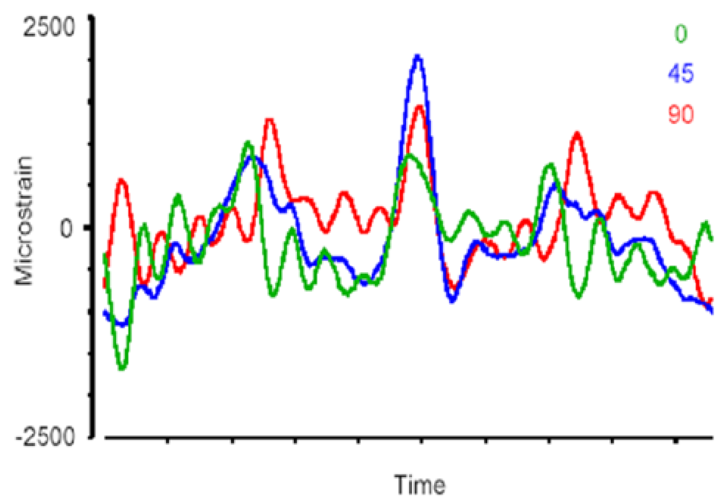
- Background
- A proposed Moment of Load Path (MLP) based non-proportional multi-axial fatigue damage model
  - Load path non-proportionality
  - Material sensitivity parameter
- Validations and applications of MLP model:
  - Structural steels
  - Aluminum alloys
  - Welded structural components
- Conclusions and contributions

Multi-axial variable amplitude stress histories

Many engineering components are subjected to multi-axial variable amplitude stress histories:



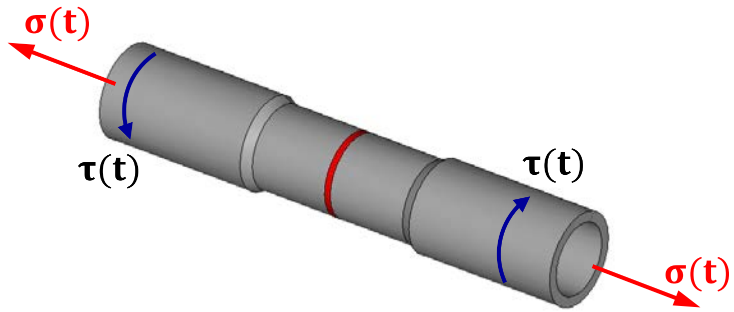
Crankshaft

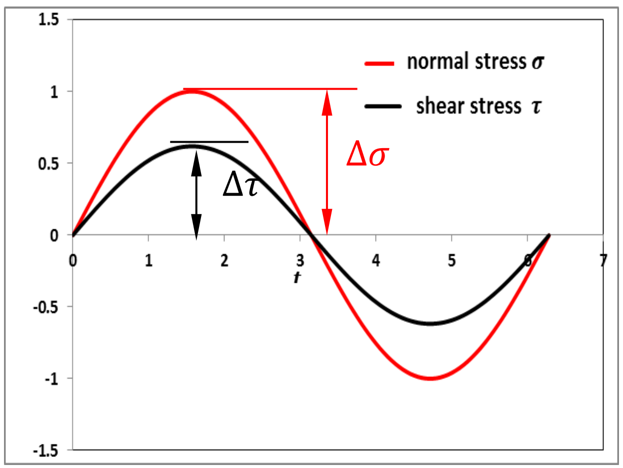


Measured microstrain along 0°, 45° and 90° directions of crankshaft

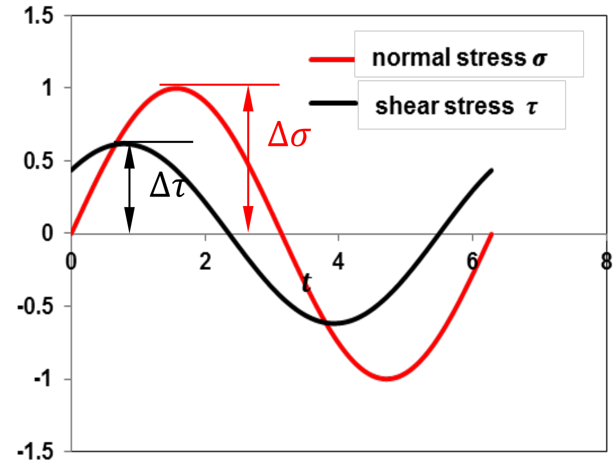
Ref: Darrell Socie, Multi-axail Fatigue, 1999

Sinusoidal in phase and out of phase loading histories





Proportional loading: in phase loading

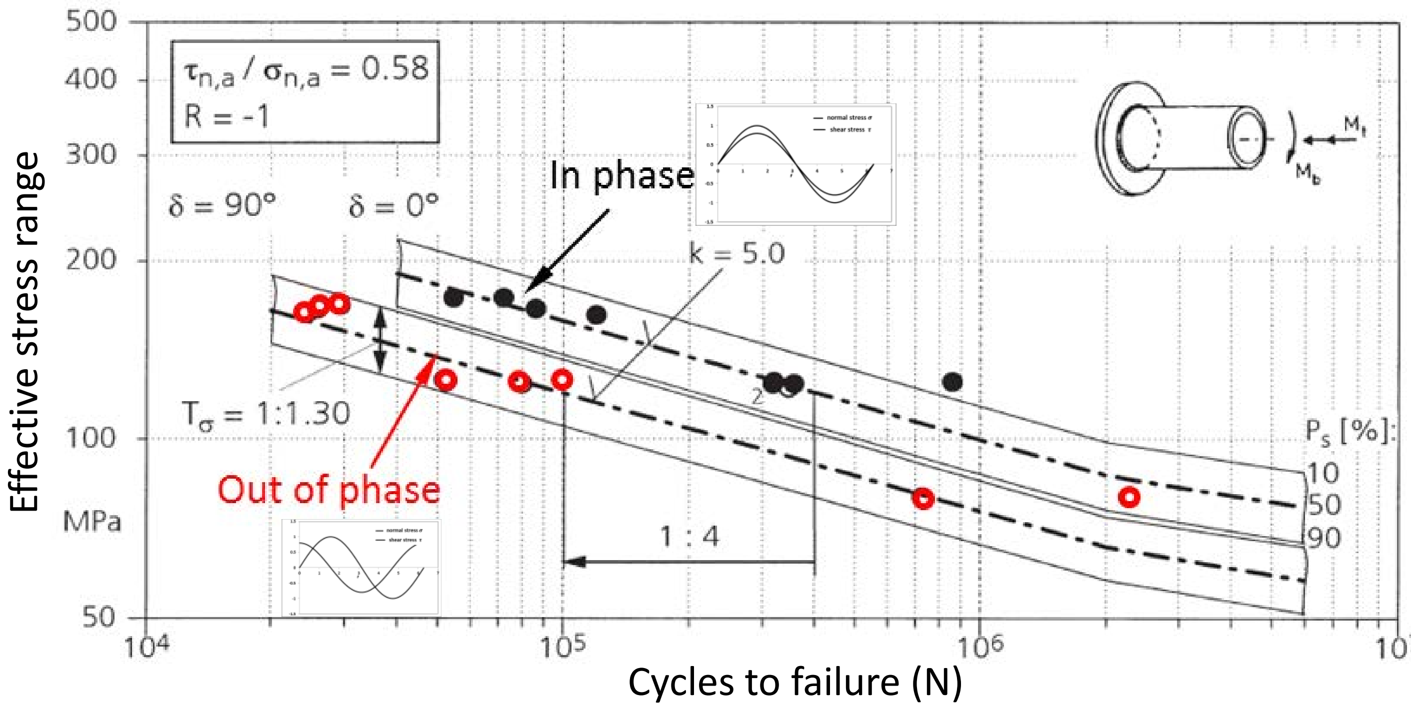


Non-proportional loading: out of phase loading

Non-proportional loading is more damaging - Steels

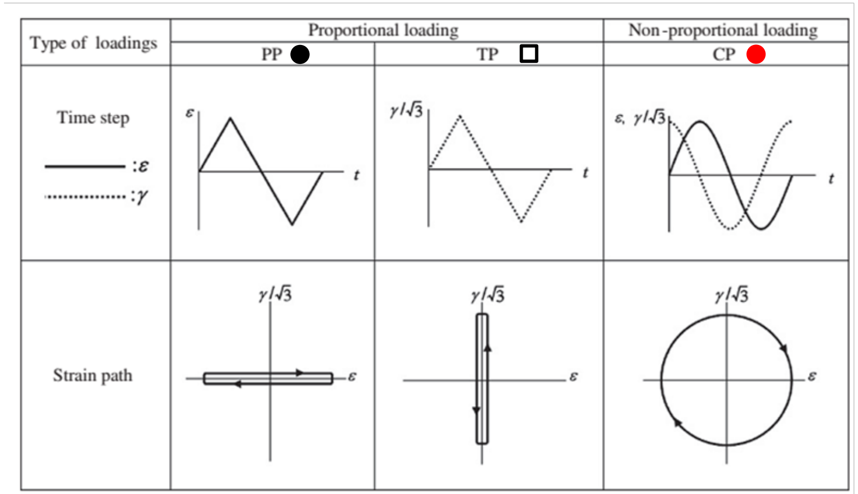
- Conventional effective stress (von Mises stress) range:

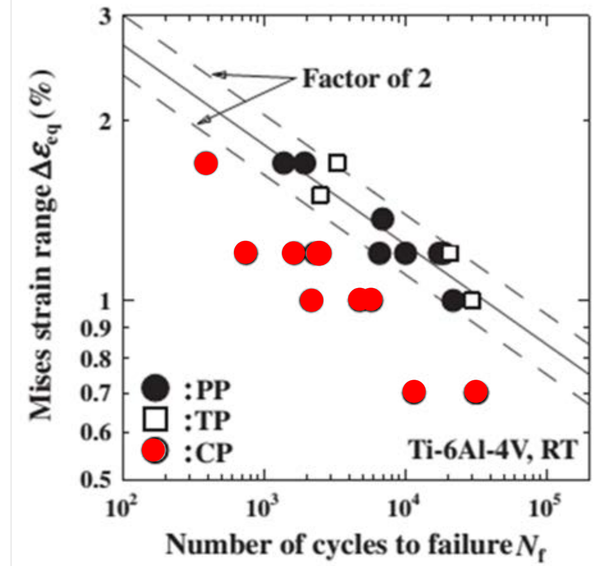
$$\Delta\sigma_e = \sqrt{\Delta\sigma_n^2 + 3\Delta\tau_n^2}$$



Ref: Sonsino, C. M., Kueppers, M. (2001).

Non-proportional loading is more damaging -Titanium alloys





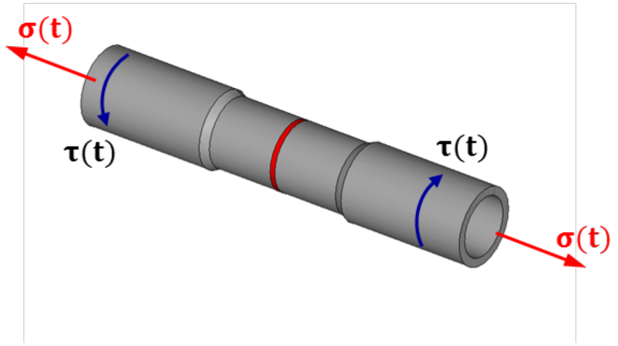
Von Mises strain range:

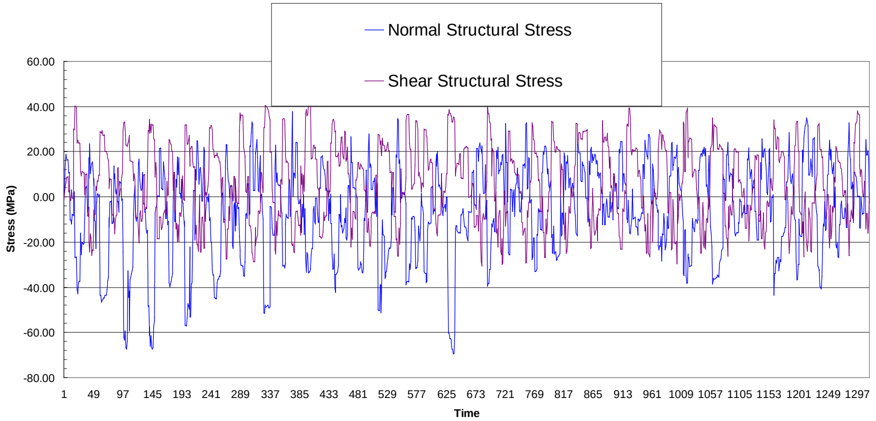
$$\Delta\varepsilon_e = \sqrt{\Delta\varepsilon^2 + 1/3\Delta\gamma^2}$$

Ref: Nakamura, H et al. (2011).

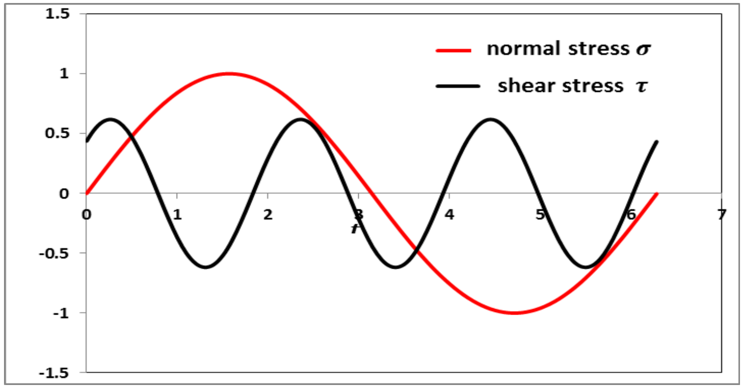
Two major research topics

- Fatigue damage parameter definition?
- Fatigue cycle definition?





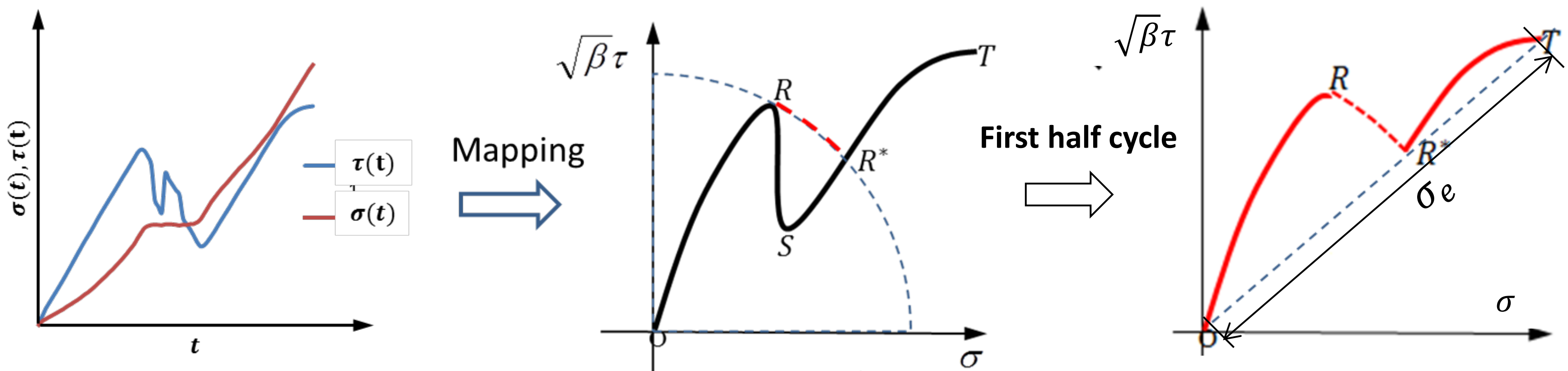
Non-proportional variable amplitude time histories



Sinusoidal wave forms (e.g. $f_b/f_t = 1/3$)

Fatigue cycle definition: PDMR multiaxial cycle counting procedure

- Path dependent maximum range (PDMR) cycle counting (Dong, Wei 2010)



σ, τ time histories

Load path

First half cycle with max range

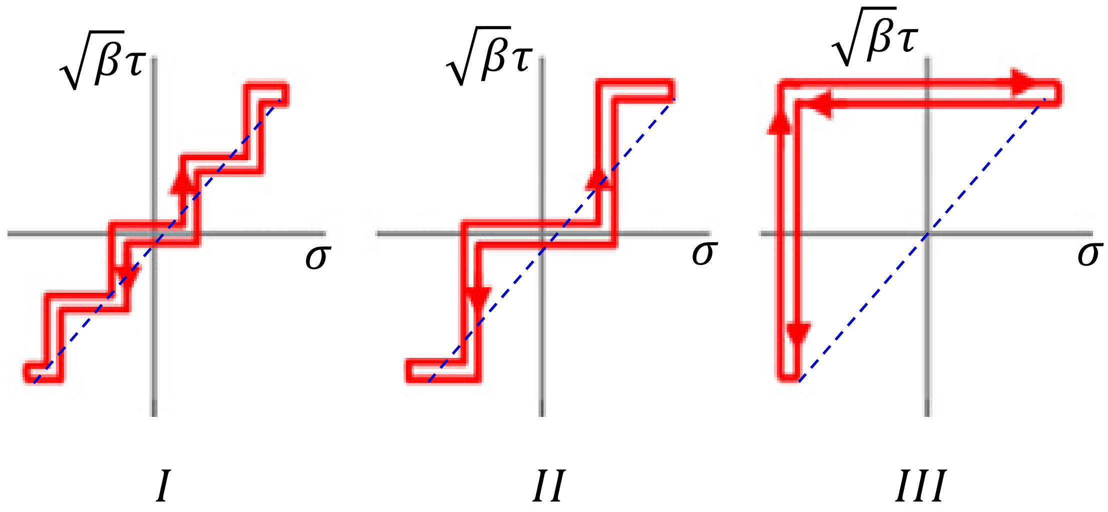
- Output:
  - Effective stress range
  - Cycle
  - Path length

PDMR procedure

- Map stress histories to $\sigma - \sqrt{\beta}\tau$ plane
- Search max stress range
- Identify monotonically increasing load path segments
- Search for remaining load paths

One limitation in path length based fatigue damage parameter

Consider tested load paths



- Path length as a fatigue damage parameter proven effective for many test cases
- One deficiency: the above three non-proportional path have the same path length but different fatigue life

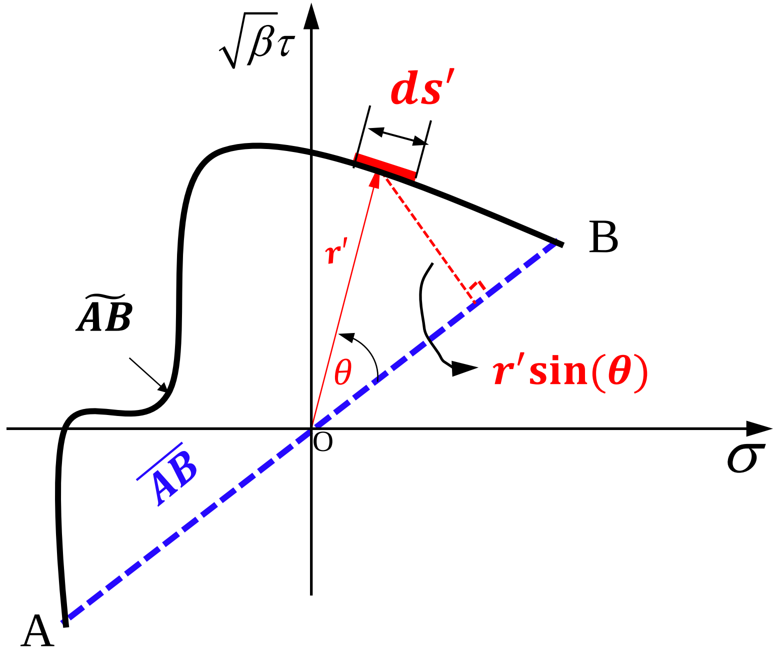
My research objectives

- Develop a generalized fatigue damage model for non-proportional variable amplitude multi-axial loading

- Perform comprehensive validations by demonstrating its ability for correlating a large amount of well-documented test data at both material and component levels

A proposed model:
Definitions and assumptions

- **Term definitions**
  - $\widetilde{AB}$: Actual non-proportional loading path after cycle counting
  - $\overline{AB}$: Reference proportional loading path of $\widetilde{AB}$



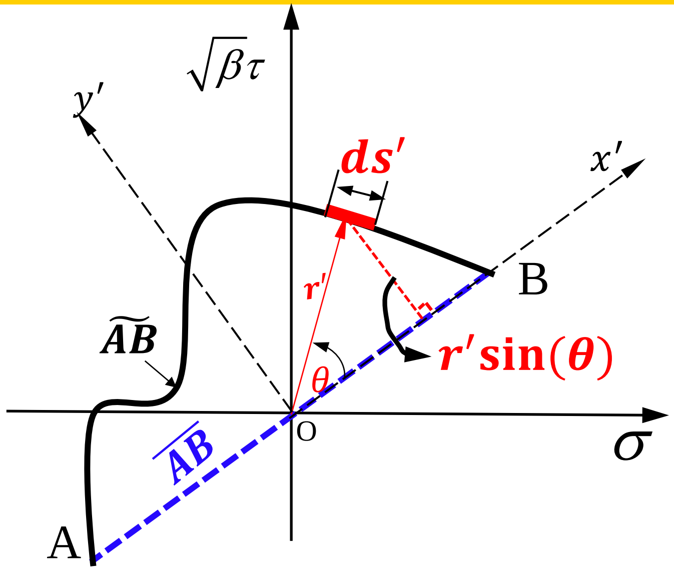
- **Hypothesis and theory:**
  - Total damage of load path $\widetilde{AB}$ can be modeled by two parts:
    - A reference (proportional) loading event, i.e., $\overline{AB}$
    - A non-proportionality measure deviating from $\overline{AB}$
  - An incremental non-proportional damage may be expressed as:

$$dD_{NP} = r'|sin\ (\theta)|ds'$$

Physical interpretation of non-proportional damage D_{NP}

$$D_{NP} = \int_{\widetilde{AB}} r' \cdot |\sin(\theta)|\, ds'$$

$$= E \int_{\widetilde{AB}} \left| p(\sigma,\tau)\underbrace{\sigma d\varepsilon}_{\text{Incremental tensile strain energy density}} + q(\sigma,\tau)\underbrace{\tau d\gamma}_{\text{Incremental shear strain energy density}} \right|$$



Dimensionless path dependent weight function:

$$p(\sigma,\tau) = -sin\left(\theta_0\right)\sqrt{1+\beta\left(\frac{d\tau}{d\sigma}\right)^2}$$

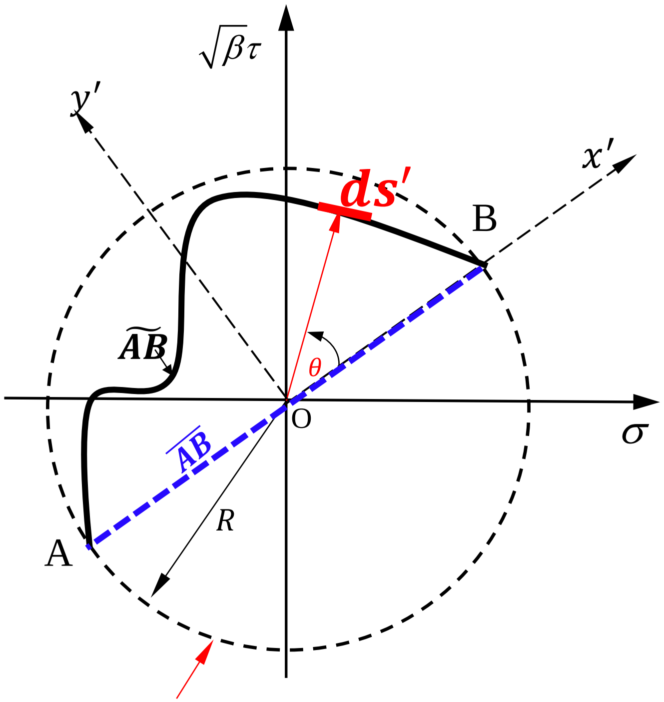
$$q(\sigma,\tau) = \frac{1}{2(1+\upsilon)}\beta cos\left(\theta_0\right)\sqrt{1+\frac{1}{\beta}\left(\frac{d\sigma}{d\tau}\right)^2}$$

- D_{NP} can be interpreted as an integral form of strain energy densities weighted by path-dependent functions

A dimensionless definition of D_{NP}: g_{NP}

Damage ratio with respect to maximum possible non-proportional load path induced damage from A to B:

$$g_{NP} = \frac{D_{NP}}{D_{Max}} = \frac{\int_{\widetilde{AB}} r'|\sin(\theta)|ds'}{\int_{\widetilde{AB}} R|\sin(\theta)|ds'} = \frac{\int_{\widetilde{AB}} r'|\sin(\theta)|ds'}{2R^2}$$



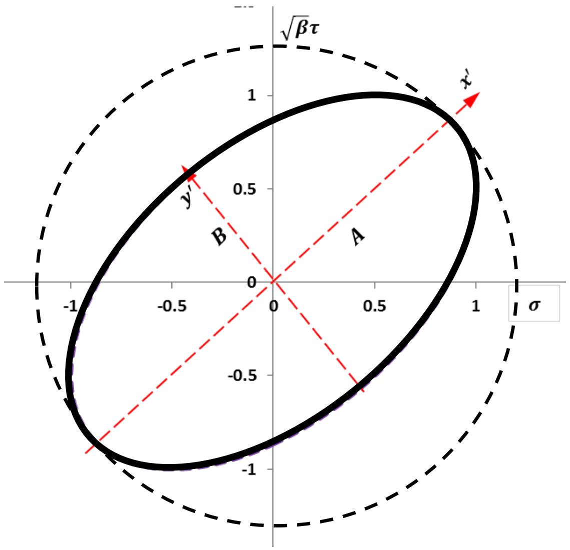
circular path: most damaging path

g_{NP}: Load path non-proportionality factor

Analytical form of g_{NP} for elliptical path

- Sinusoidal bending and torsion with a phase shift:

$$\sigma = \sigma_0 \sin(\theta)$$
$$\tau = \tau_0 \sin(\theta - \delta)$$



- Semi long axis and semi short axis length:

$$A = \sqrt{\frac{2\beta\sigma_0^2\tau_0^2\sin^2\delta}{(\sigma_0^2+\beta\tau_0^2)-\sqrt{(\sigma_0^2+\beta\tau_0^2)^2-4\beta\sigma_0^2\tau_0^2\sin^2\delta}}}$$

$$B = \sqrt{\frac{2\beta\sigma_0^2\tau_0^2\sin^2\delta}{(\sigma_0^2+\beta\tau_0^2)+\sqrt{(\sigma_0^2+\beta\tau_0^2)^2-4\beta\sigma_0^2\tau_0^2\sin^2\delta}}}$$

- Closed-form solution of g_{NP}:

$$g_{NP}(\sigma_0,\tau_0,\delta) = \frac{\eta}{2}\left(\eta + \frac{\arcsin(\sqrt{1-\eta^2})}{\sqrt{1-\eta^2}}\right), \qquad where\ \eta = \frac{B}{A}$$

MLP-based equivalent stress and strain definitions for test data correlation

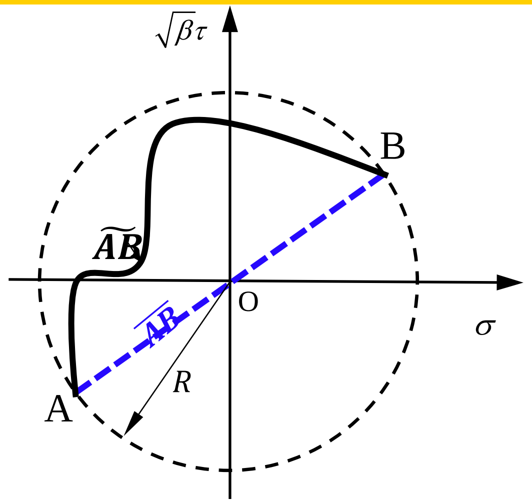
- MLP based equivalent stress

$$\Delta\sigma_{NP} = \Delta\sigma_{\overline{AB}}(1 + g_{NP})$$

Or,

$$\Delta\sigma_{NP} = \Delta\sigma_{\overline{AB}}(1 + \alpha \cdot g_{NP})$$

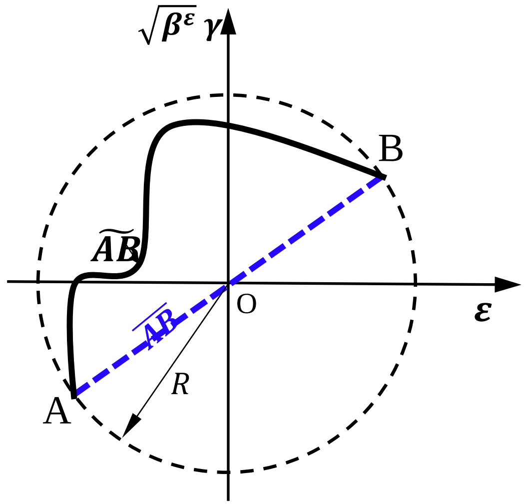
  - $\Delta\sigma_{NP}$: **MLP based equivalent stress range**.
  - $\Delta\sigma_{\overline{AB}}$ is effective stress range of $\overline{AB}$
  - α is a material sensitivity parameter



- MLP based equivalent strain

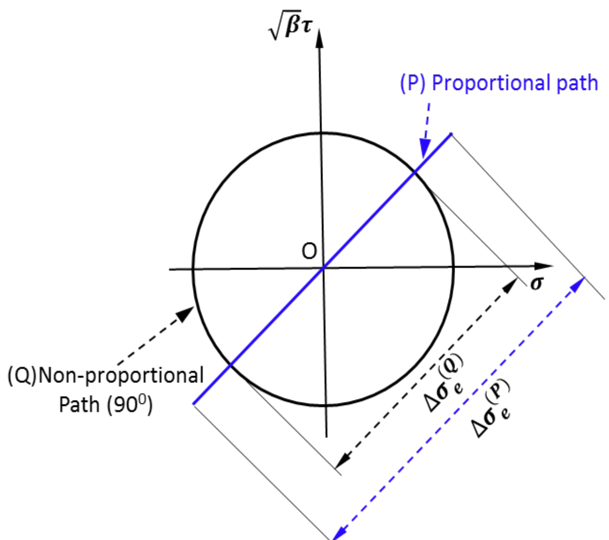
$$\Delta\varepsilon_{NP} = \Delta\varepsilon_{\overline{AB}}(1 + \alpha^{\varepsilon} \cdot g_{NP})$$

  - $\Delta\varepsilon_{NP}$: **MLP based equivalent strain range**.

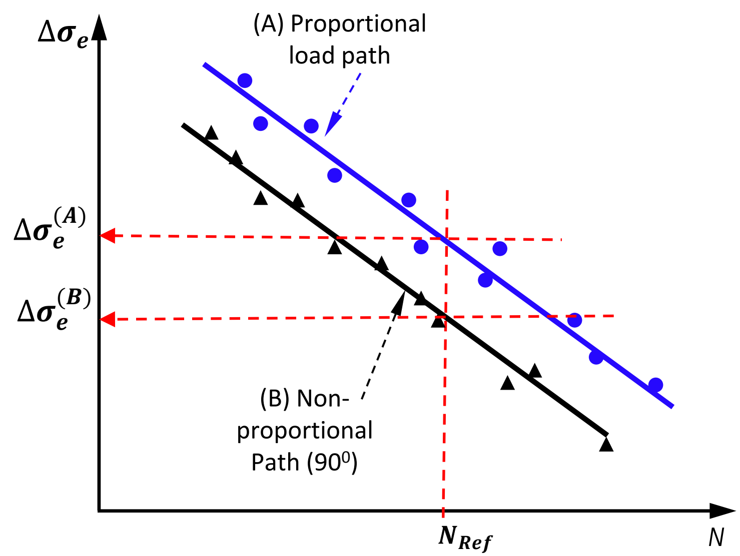


Procedure for determining material sensitivity parameter α to load path non-proportionality

- Two groups of fatigue tests
  - Proportional(in phase) test
  - 90° out-of-phase test



(a)



For prop path A: $\Delta\sigma_{NP}^{A} = \Delta\sigma_e^{(A)}(1 + \alpha \cdot g_{NP}) = \Delta\sigma_e^{(A)}$ 0

For Non-prop path B: $\Delta\sigma_{NP}^{B} = \Delta\sigma_e^{(B)}(1 + \alpha \cdot g_{NP})$ 1

$$\alpha = \frac{\Delta\sigma_e^{(A)}}{\Delta\sigma_e^{(B)}} - 1$$

Validations and applications

MLP based fatigue model

1

Various steels (Mei, Dong, IJF, 2016)

- Comparison with path length based parameter

2

Various aluminum alloys (Mei, Dong, IJF, 2017)

- Seven groups of test data examined
- Connection of Material sensitivity and ductility

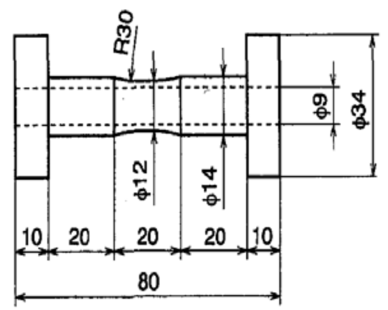
3

Comprehensive collection on weldment data (Mei, Dong, IJF, 2017)

- Comparison with Codes and standards
- PDMR cycle counting demonstrated

Application of MLP model in test data correlation - 304 steel (Itoh et al, 1995)

- Test conditions:
  - Strain controlled test
  - Thirteen load cases tested



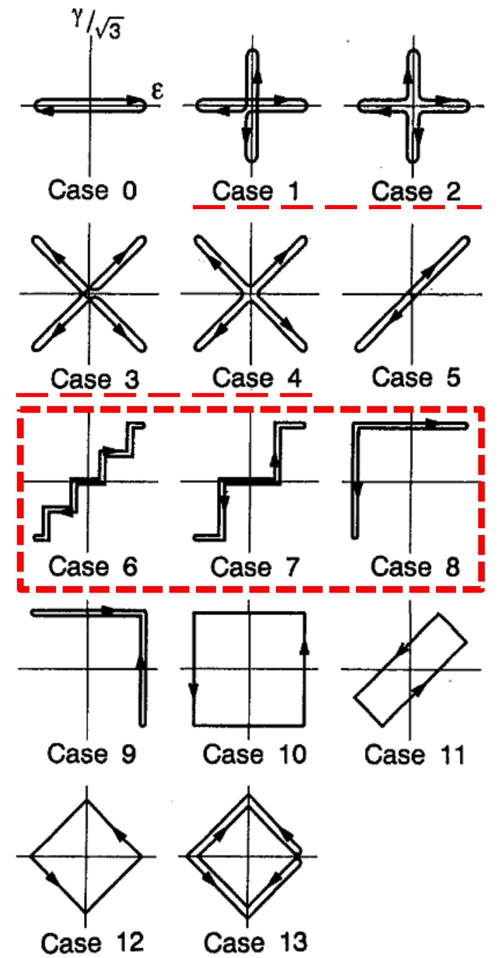
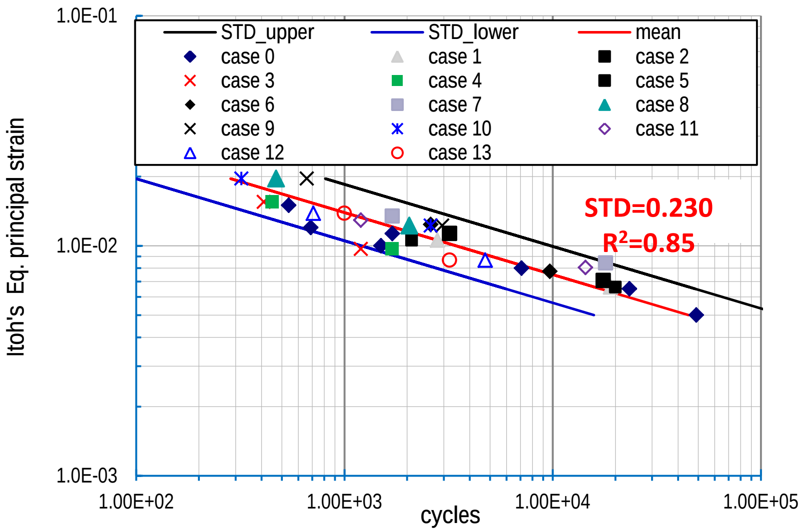


Fig. 2 Proportional and nonproportional loading paths

- Issues with Itoh et al.'s principal strain approach:
  - No clearly defined fatigue cycle
  - Empirical parameters assumed to be dependent on material hardening behavior

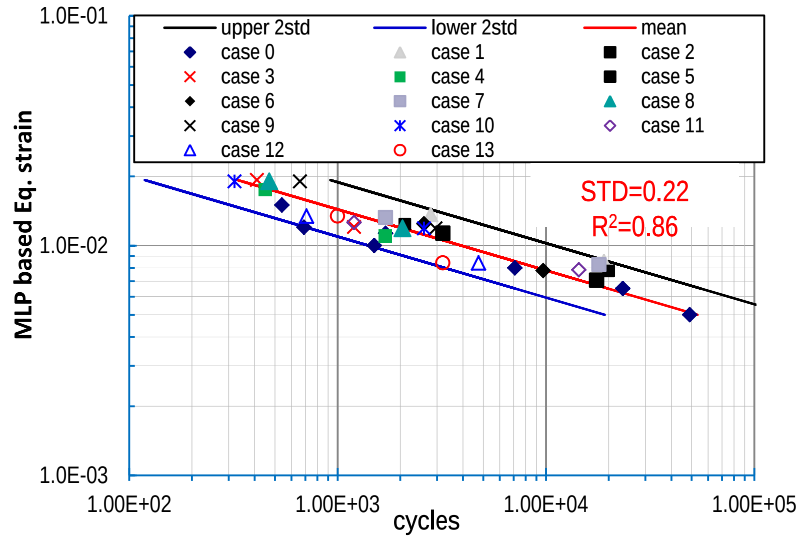
Application of MLP model to 304 steels
-Itoh et al.'s low cycle fatigue data ($\alpha = 1$)

- Principal strain based correlation (Itoh et al.)



- Hardening parameters used
- No proper cycle definition

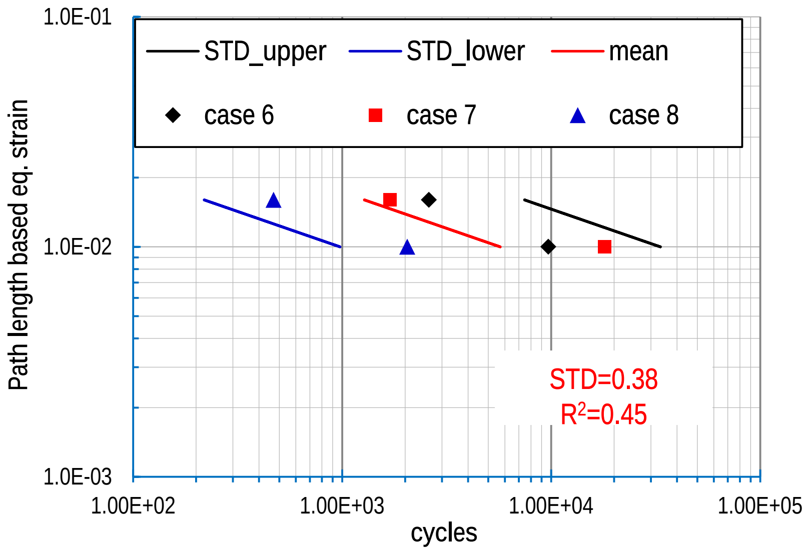
- MLP based correlation

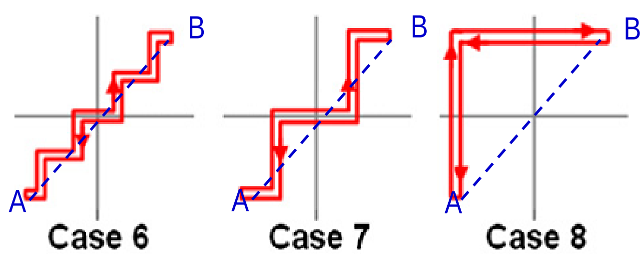


- No material constant used
- PDMR based cycle counting
- Still achieved a similar or improved correlation

Application of MLP model to 304 steels
-Itoh et al.'s low cycle fatigue data ($\alpha = 1$)

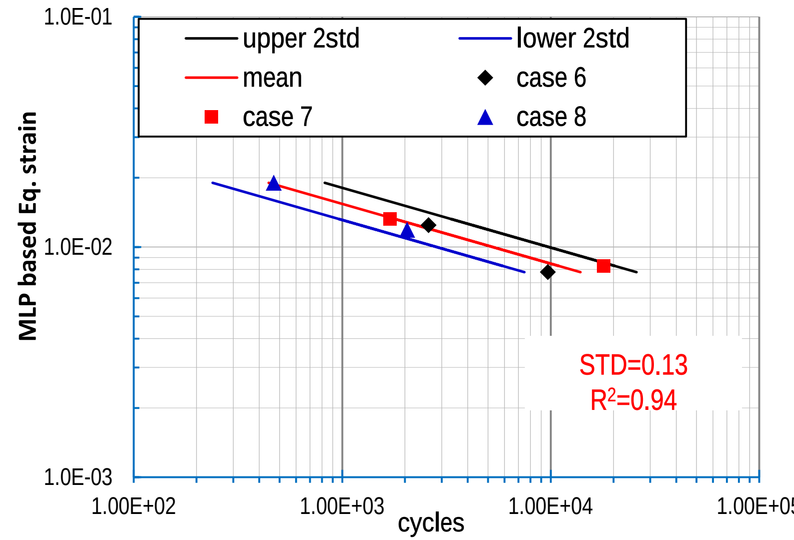
- Path length based correlation for Cases 6-8

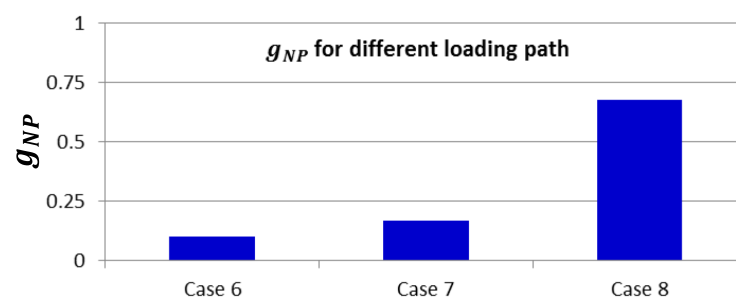




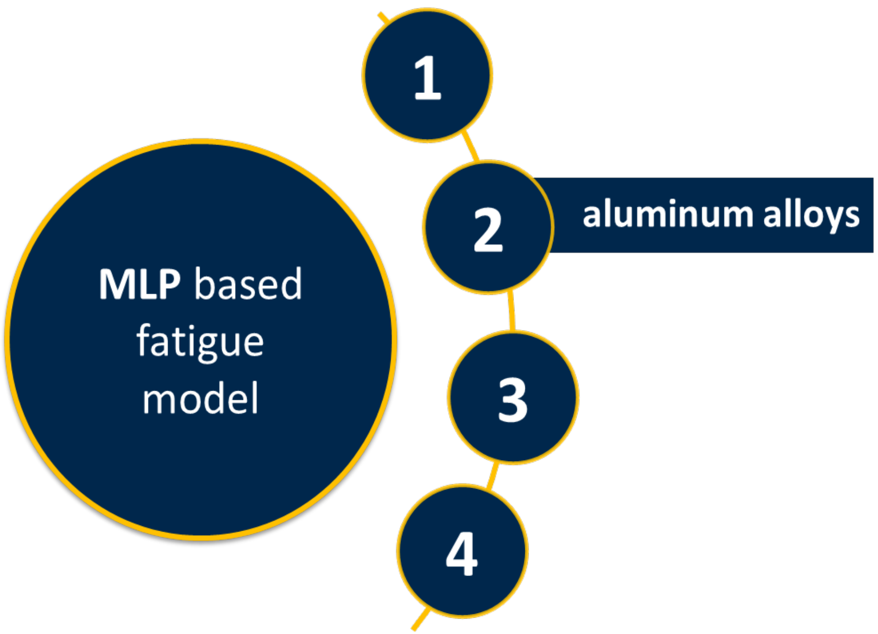
Same path length

- MLP based correlation for Cases 6-8





Application of MLP model in test data correlation - aluminum alloys



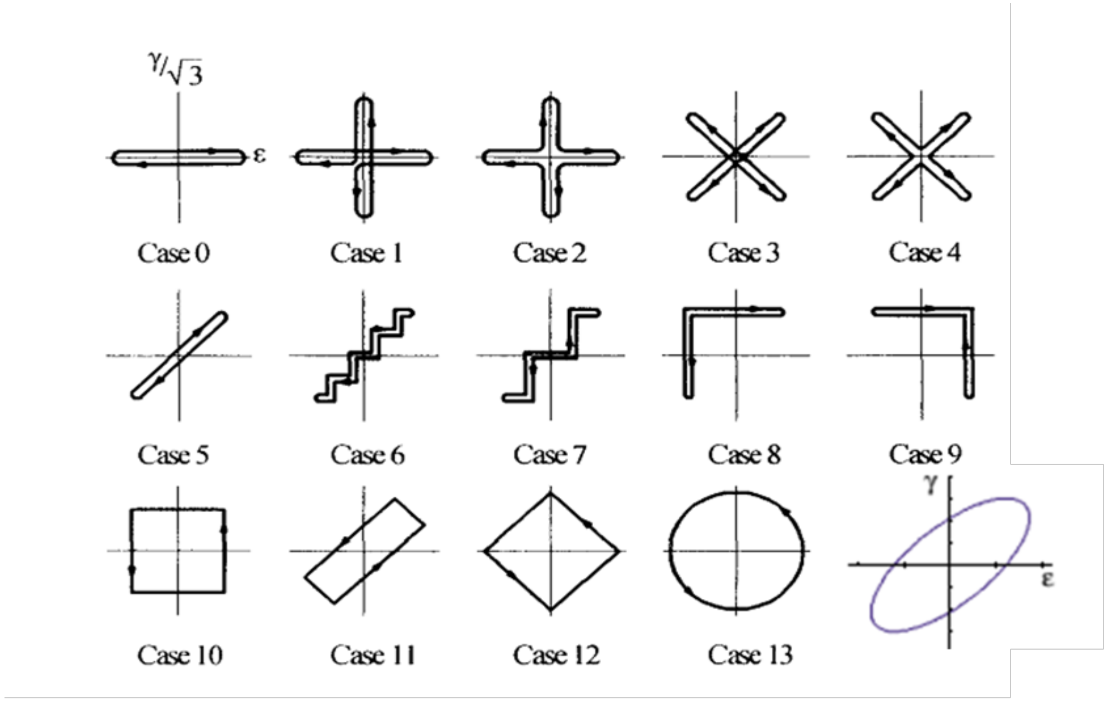
- Case 1: 2024-T4 alloy (Xia, Yao. 2013)
- Case 2: 2024-T4 alloy (Wang et al. 2011)

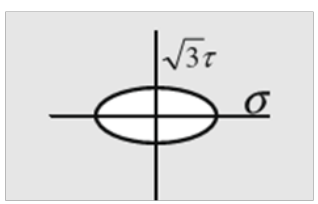
- Case 3: 5083 alloy, Hoffmeyer et al . (2006)

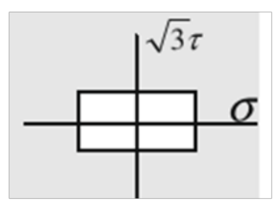
- Case 4: 6061 alloy, Itoh et al. (1997)
- Case 5: 6082-T6 alloy, Susmel, Petrone(2003)

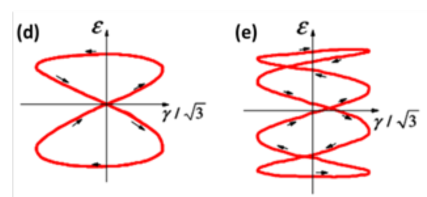
- Case 6 : 7075-T651 alloy, Zhao, Jiang. (2008)
- Case 7 : 7075-T6 alloy, Zamrick and Frishmuth (1973)

Representative load paths used by those researchers





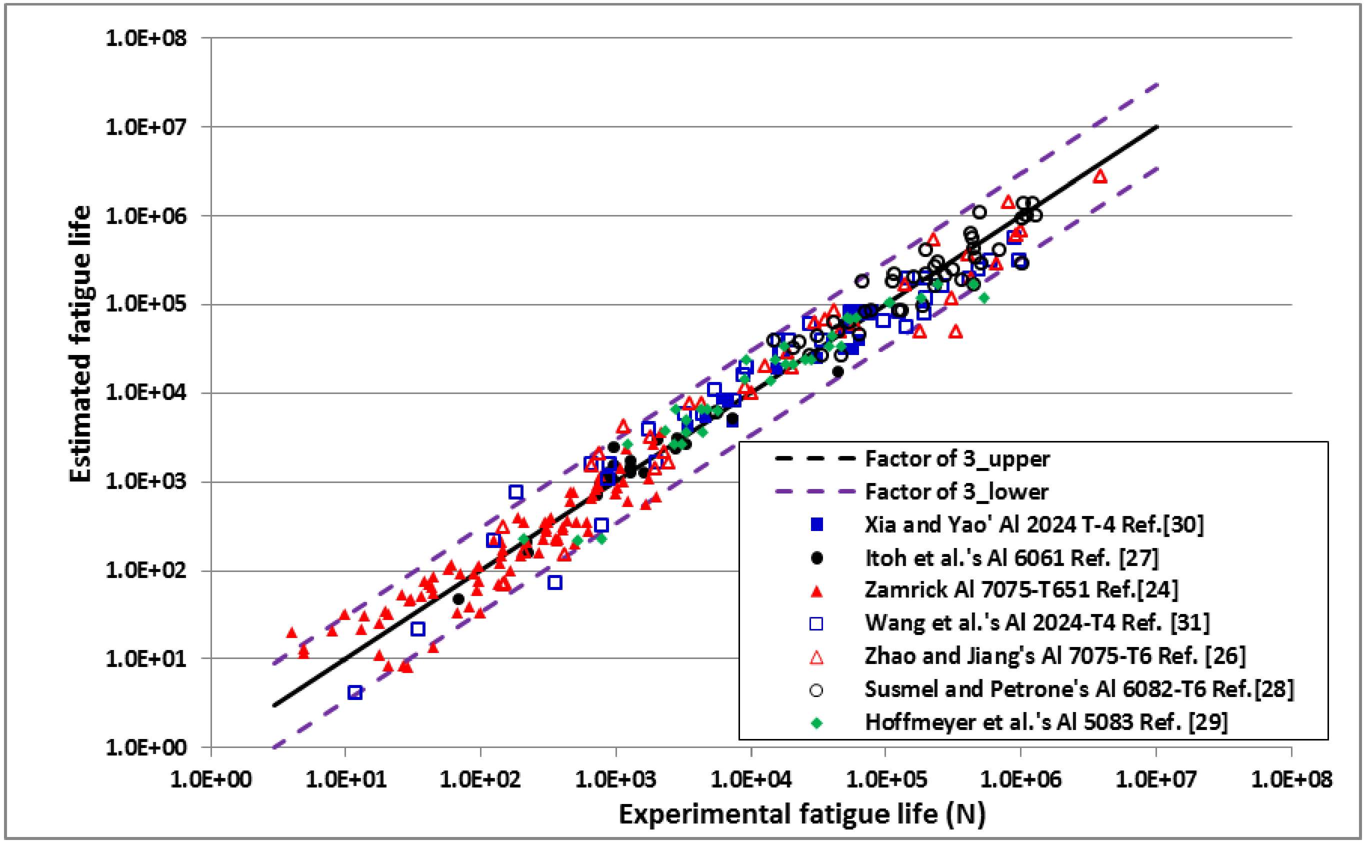




Data analysis procedure

- PDMR cycle counting: output counted cycles, stress ranges ($\Delta\sigma_e$), and load path
- Load path non-proportionality (g_{NP})
- Determination of material sensitivity parameter (α)
- Calculate MLP based equivalent stress
- Compare MLP model estimated lives versus actual test lives

Data analysis results: MLP based prediction vs. experimental results



(Ref: Mei, Dong, IJF, 2017)

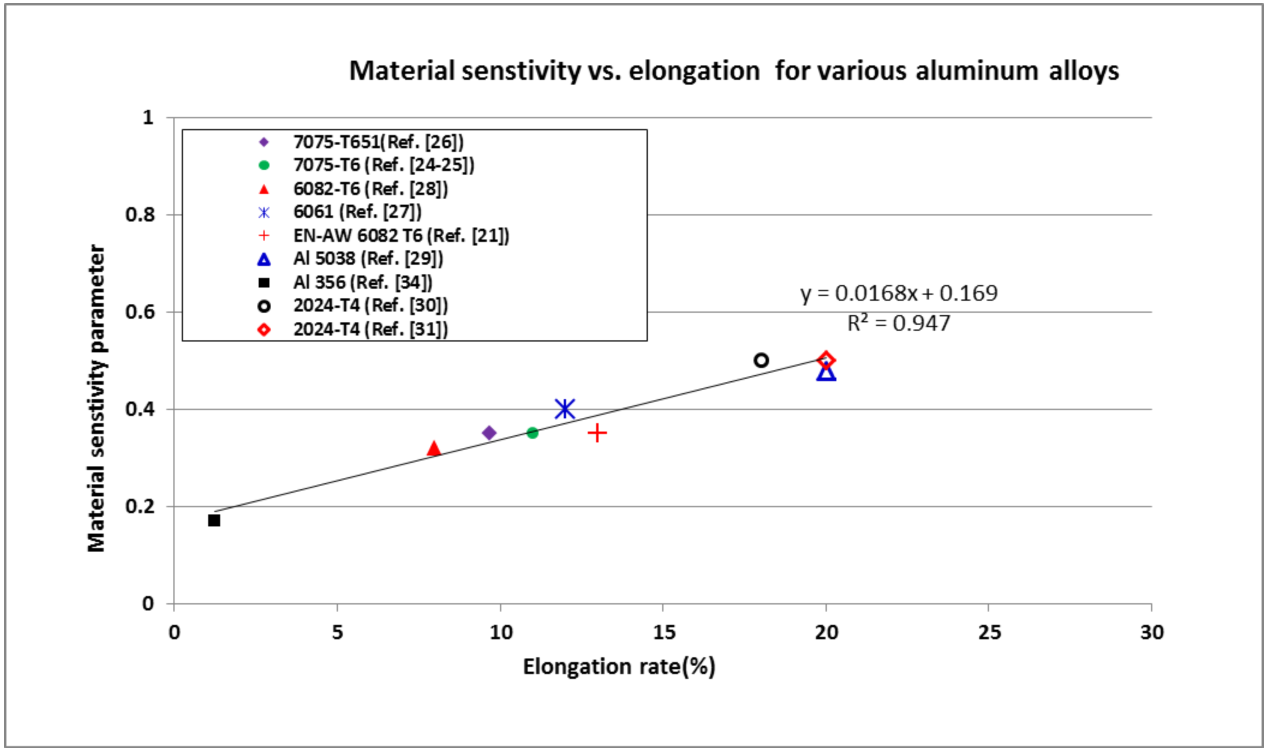
Comparison of calculated material sensitivity parameter for all aluminum alloys

Aluminum alloys	α or α^{ε}	Ductility (Elongation, %)
6082-T6 [28]	α=0.35	8
7075-T651 [24-25]	α^{ε}=0.35	9.7
7075-T6 [26]	α^{ε}=0.35	11
6061-T6 [27]	α^{ε}=0.4	12
5083 [29]	α^{ε}=0.48	20
2024-T4 [30]	α=0.5	18
2024-T4 [31]	α^{ε}=0.5	20

- Some observations:
  - Aluminum alloys is less sensitive to non-proportional loading than structural steels
  - As material ductility increases (from 7000 series to 2000 series), material sensitivity to non-proportional loading also increases

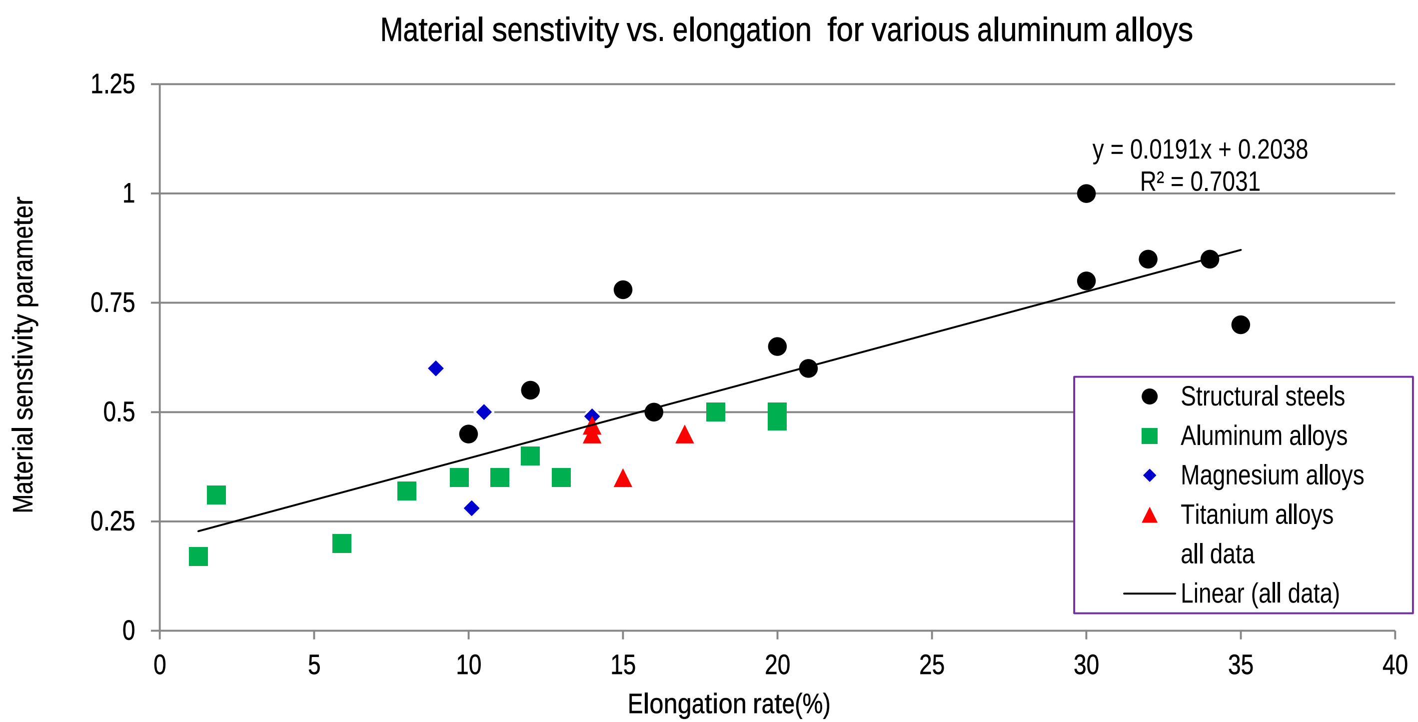
An approximately linear relation between material sensitivity vs. ductility

All aluminum alloys investigated



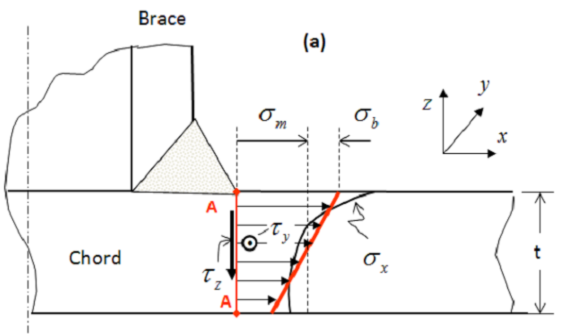
(Ref: Mei, Dong, IJF, 2017)

An approximately linear relation between material sensitivity vs. ductility, including other materials

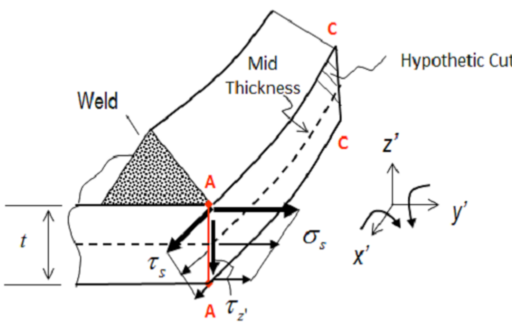


Applications to welded structural components
- Stress definition

- Traction based structural stress
  - Three traction stress component exposed by a hypothetical cut
  - In equilibrium with applied far field stress
  - Mesh insensitivity

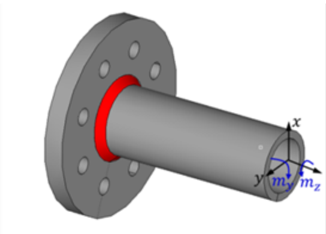


- Only normal structural stress (σ_s) and in-plane (τ_s) considered

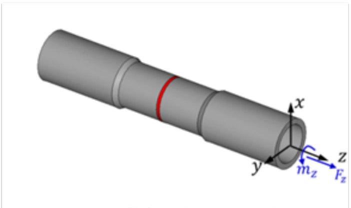


Applications for evaluation of welded structural components

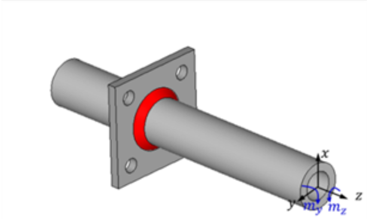
- Residual stress relieved
- As welded



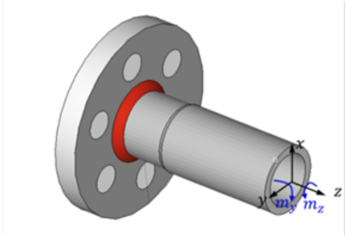
(a) Tube-to-flange joint
(Sonsino and Kuepper[24])



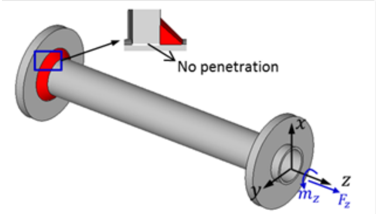
(b) Tube-to-tube joint
(Sonsino and Łagoda [25])



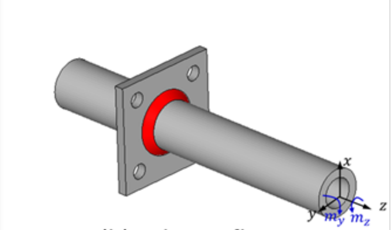
(c) Tube-to-flange joint
(Siljander et al.[26])



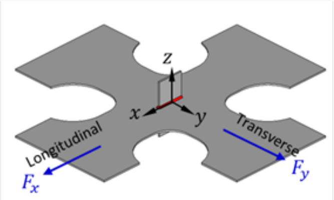
(d) Tube-to-flange joint
(Yousefi et al.[27] ;Witt and Zenner [28])



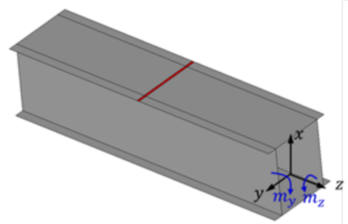
(a) Tube-to-flange joint
(Razmjoo [29])



(b) Tube-to-flange joint
(Yung and Lawrence [30])



(c) Fillet welded gusset
(Takahashi et. al [31,32])



(d) Transverse welded box beam
(Dahle et al. [33])

Ref: J. Mei, P. Dong. An equivalent stress parameter for modeling multi-axial fatigue of welds. I.J.F., 2017

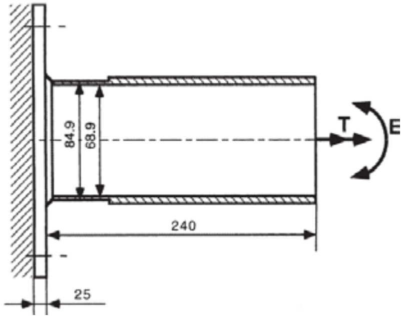
Stress concentration calculation results – different stress definitions

Multi-axial fatigue test	Structural stress SCF - normal stress $(\sigma_m + \sigma_b)/\sigma_n$	Structural stress SCF - shear stress $(\tau_m + \tau_b)/\tau_n$	Hot spot SCF - normal stress σ_h/σ_n	Hot spot SCF - shear stress τ_h/τ_n
Sonsino and Kueppers[27]	1.72	1.1	2.2	1.1
Sonsino and **Łagoda** [28]	1.00	1	1.0	1.0
Siljander et al.[29]	1.42	1.1	1.25	1.1
Yousefi et al.[30]	1.69	1.1	1.37	1.1
Witt and Zenner [31]	1.69	1.1	1.37	1.1
Razmjoo [32]	1.90	1.1	1.4	1.1
Yung and Lawrence [33]	1.42	1.1	1.25	1.1
Takahashi et. al [34,35]	1.95	0	1.73	0
Dahle et al. [36]	1.00	1	1.0	1.0

Ref: J. Mei, P. Dong. An equivalent stress parameter for modeling multi-axial fatigue of welds. I.J.F., 2017

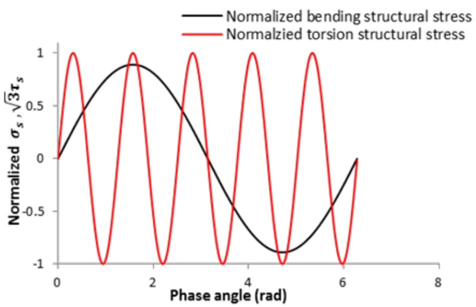
Example: Analysis of Yousfei's test data(2001)
- Description of tests

- Five test conditions :
  - Pure bending
  - Pure torsion
  - In phase
  - Out of phase
  - Asynchronous loading



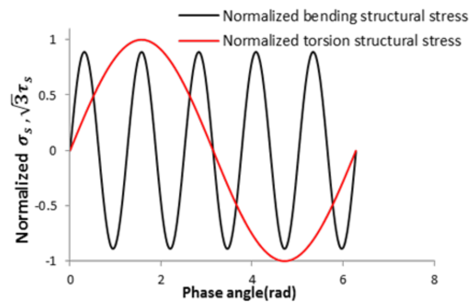
- Authors' cycle definitions:

Asynchronous loading:



(a) Time history of bending and torsion stress (f_b/f_t=1/5)

$f_b/f_t = 1/5$
Assumed cycles: 5

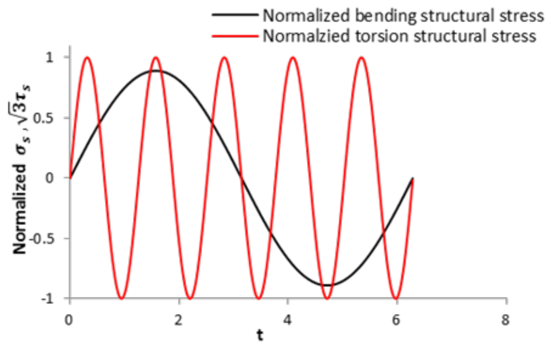


(b) Time history of bending and torsion stress (f_b/f_t=5)

$f_b/f_t = 5$
Assumed cycles: 5

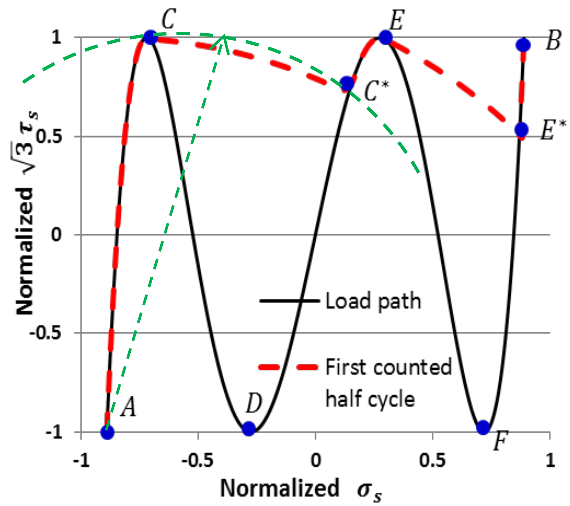
Analysis of Yousfei's test data
- PDMR cycle counting

- $f_b/f_t = 1/5$ for example



(a) Time history of bending and torsion stress (f_b/f_t=1/5)

Stress components load history



Black line: loading path
Red dashed line: half cycle with max range

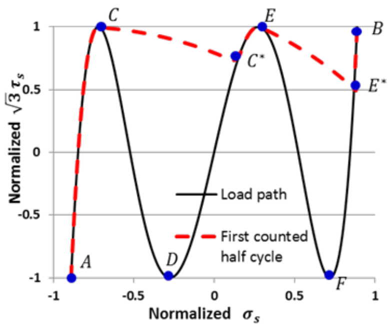
- Step1: Search max range (AB)
- Step2: Identify load path with monotonically increasing range until max range is reached. count as one half cycle

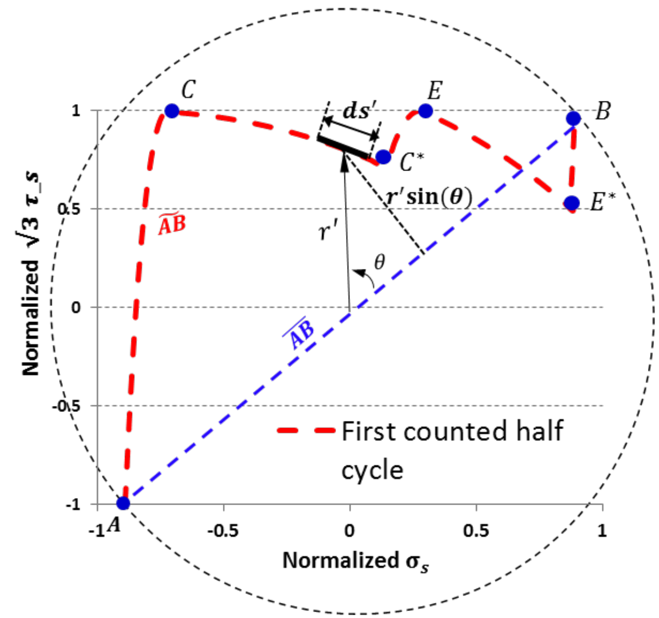
- PDMR based cycles:

Counted cycle	Range	Load path
0.5	AB	AC-CC*-C*E-EE*-E*B
0.5	CD	CD
0.5	EF	EF
0.5	DC*	DC*
0.5	FE*	FE*

Analysis of Yousfei's test data

- MLP based equivalent stress calculation



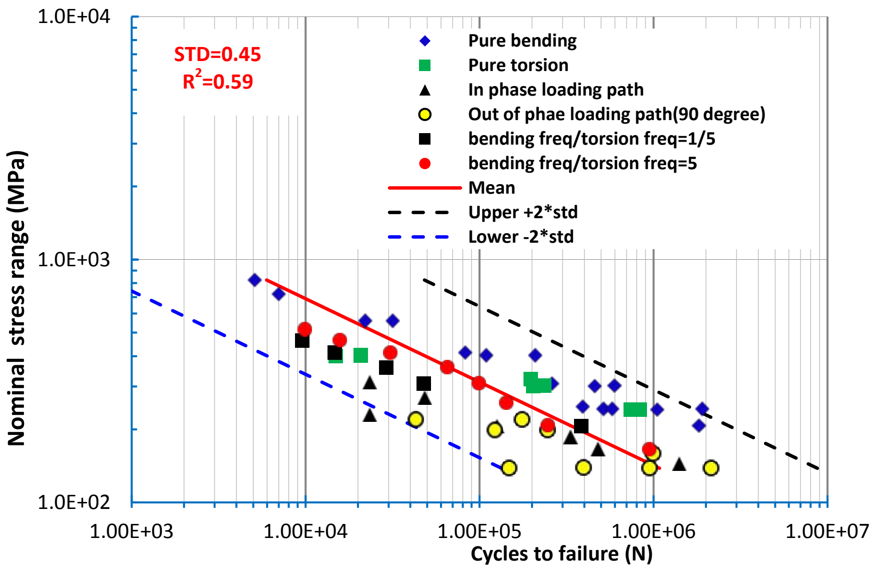


MLP-based equivalent stress range results

ID (i)	Counted cycles (N_i)	Effective stress range ($\Delta\sigma_{e,i}$)		Load path	Non-proportional factor($g_{NP,i}$)	MLP-based equivalent stress ($\Delta\sigma_{NP,i}$)
		Stress range	Range value			
1	1	AB	2.67	AC-CC*-C*E-EE*-E*B	0.66	4.4322
2	1	CD	2.04	CD	0.1	2.244
3	1	EF	2.03	EF	0.1	2.233
4	1	DC*	1.8	DC*	0.09	1.962
5	1	FE*	1.45	FE*	0.08	1.566

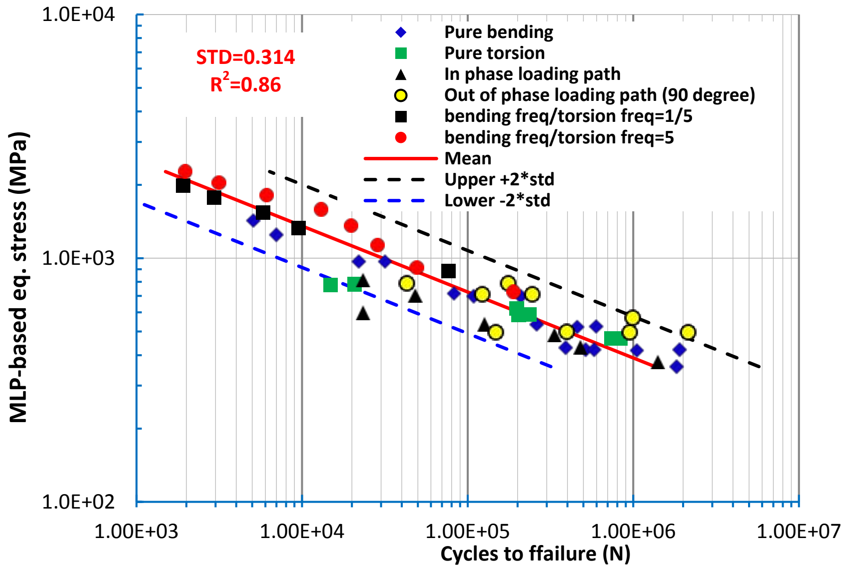
Analysis of Yousfei's test data
- Data correlation and comparison

- Nominal stress based data correlation



  - Large scatter band

- MLP based data correlation



  - Very good correlation

Further Extension of MLP model for fatigue design and evaluation of welded components

- Original equivalent structural stress (ASME, 2007)

$$\Delta S_S = \frac{\Delta\sigma_e}{t^{*\frac{2-m}{2m}} I(r)^{\frac{1}{m}}}$$

- New equivalent stress parameter proposed

$$\Delta S_{NP}^R = \frac{\boldsymbol{\Delta\sigma_{NP}}}{(1-R)^{\frac{2}{m}} t^{*\frac{2-m}{2m}} I(r_e)^{\frac{1}{m}}}$$

$$r_e = \frac{\sqrt{\Delta\sigma_b^2 + \beta\Delta\tau_b^2}}{\sqrt{\Delta\sigma_m^2 + \beta\Delta\tau_m^2} + |\Delta\sigma_b|}$$

- Methods used by current Codes and Standards
  - Eurocode 3 :

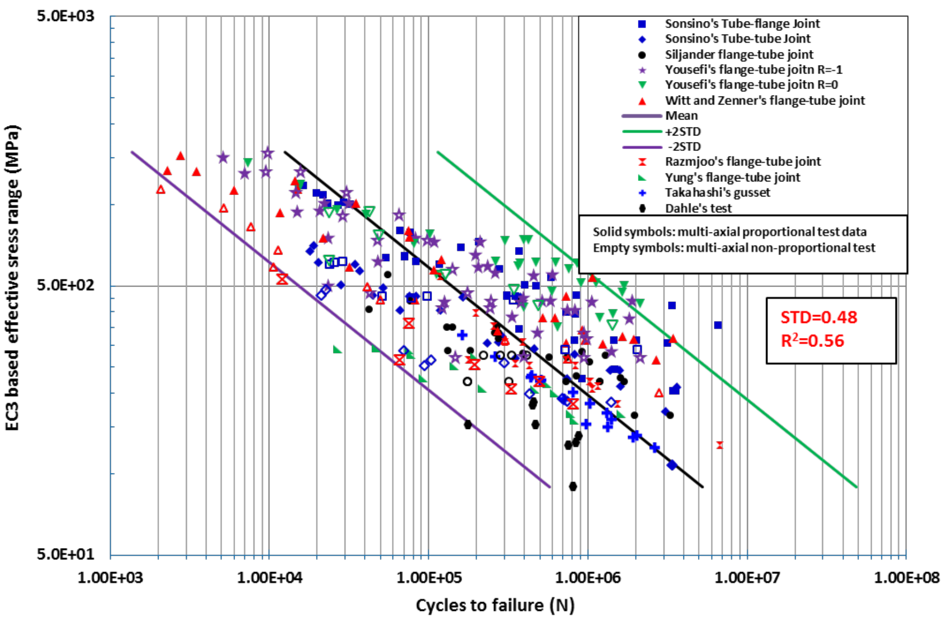
$$\Delta\sigma_{EC} = \sqrt[3]{(\Delta\sigma_h)^3 + k_{EC}\,(\Delta\tau_h)^5} \qquad k_{EC} = \Delta\sigma_f{}^3/\Delta\tau_f{}^5$$

  - IIW :

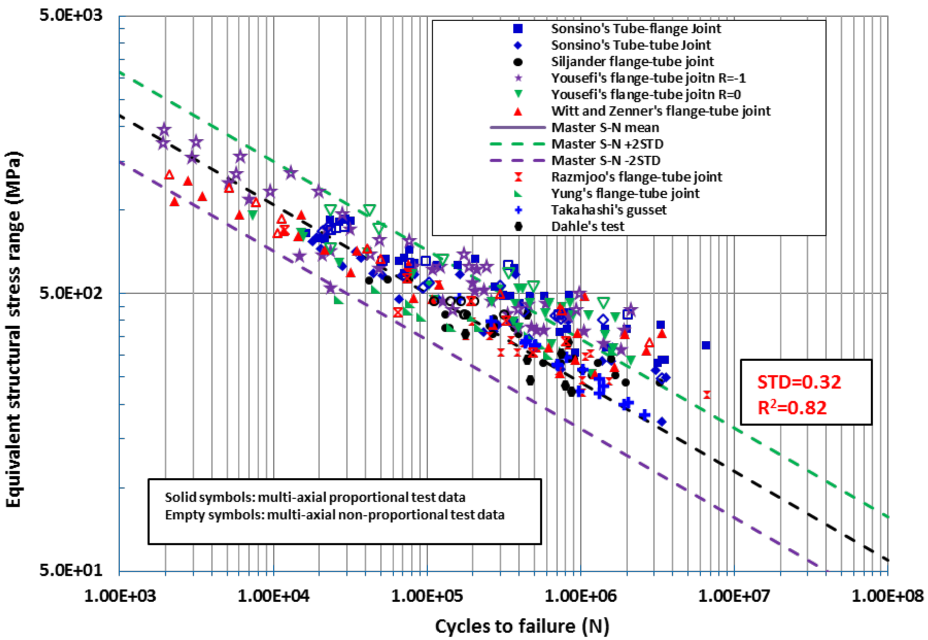
$$\Delta\sigma_{IIW} = \frac{1}{\sqrt{D_{IIW}}}\sqrt[2]{(\Delta\sigma_h)^2 + k_{IIW}(\Delta\tau_h)^2} \qquad k_{IIW} = \Delta\sigma_f{}^2/\Delta\tau_f{}^2$$

Correlation of structural component test data: Eurocode 3 (EC3) versus MLP model

- Eurocode 3 based correlation

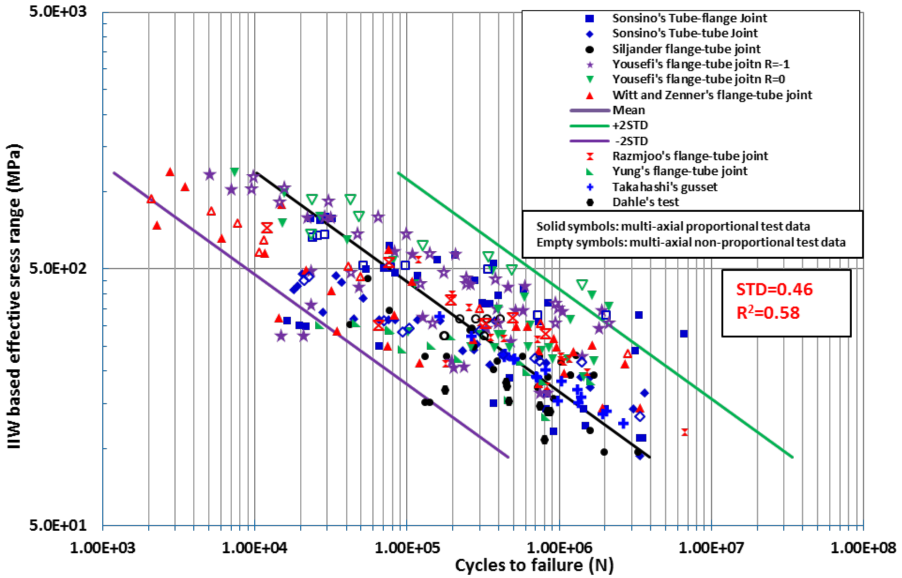


- New equivalent stress based correlation

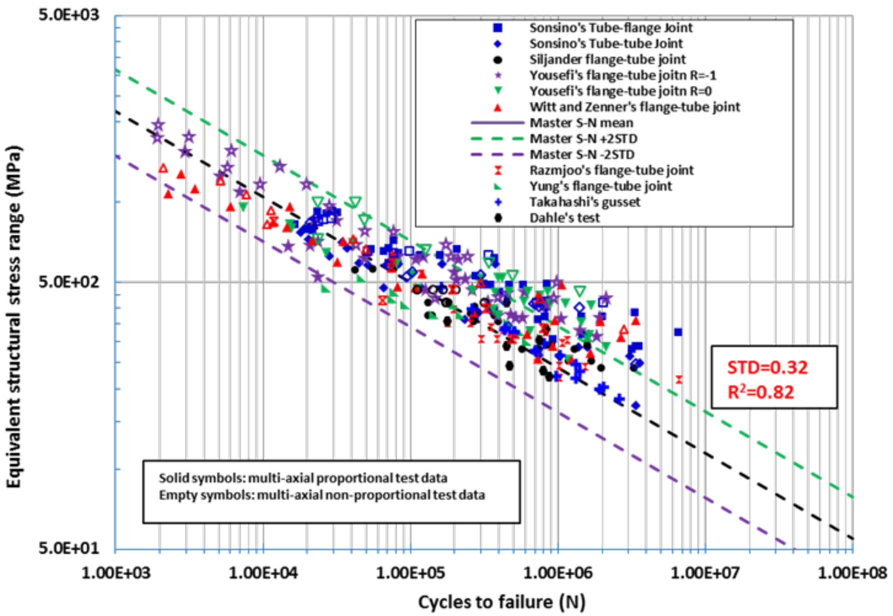


Correlation of structural component test data: IIW method versus MLP model

- IIW based effective stress



- Proposed equivalent stress



Main Contributions – Part I

- Developed a non-proportional fatigue damage model
  - non-proportionality factor (g_{NP})
  - material sensitivity parameter (α)
- Validated for applications in $\sigma - \sqrt{\beta}\tau$ plane, $\varepsilon - \sqrt{\beta^{\varepsilon}}\gamma$ plane
- Integrated in PDMR cycle counting procedure
- Validated using a large amount of test data
  - structural steels
  - aluminum alloys
  - Welded structural components

Main Contributions – Part II

- Identified a quantitative relationship between material sensitivity and ductility
- Implemented MLP based equivalent stress parameter for performing fatigue evaluation of welded structural components

$$\Delta S_{NP}^{R} = \frac{\Delta\sigma_{NP}}{(1-R)^{\frac{2}{m}}\, t^{*\frac{2-m}{2m}}\, I(r_e)^{\frac{1}{m}}}$$

- Demonstrated its effectiveness by comparing current codes and standard using structural test data

Q &A

Thank you!

Back up slides:

Why both stress and strain planes can be used in MLP model:

Experimental support:

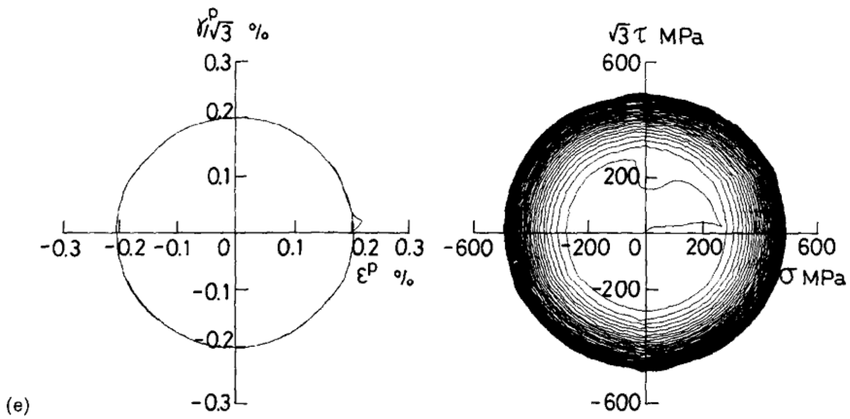
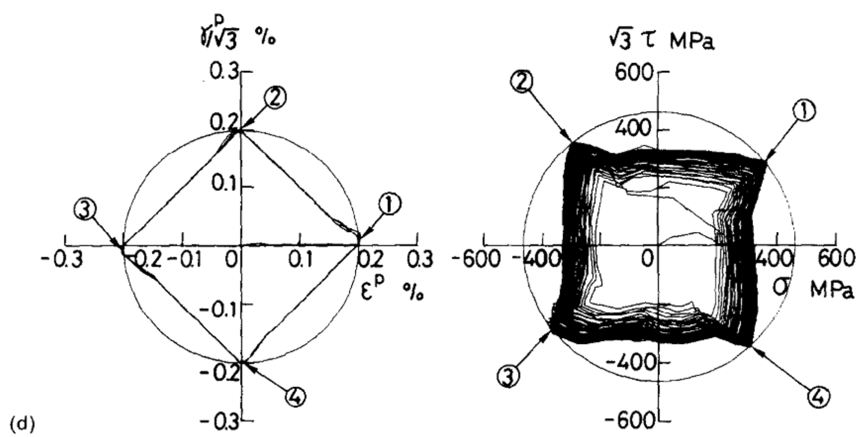


FIG. 3 (*continued*).



Why both stress and strain planes can be used in MLP model:

Experimental support:

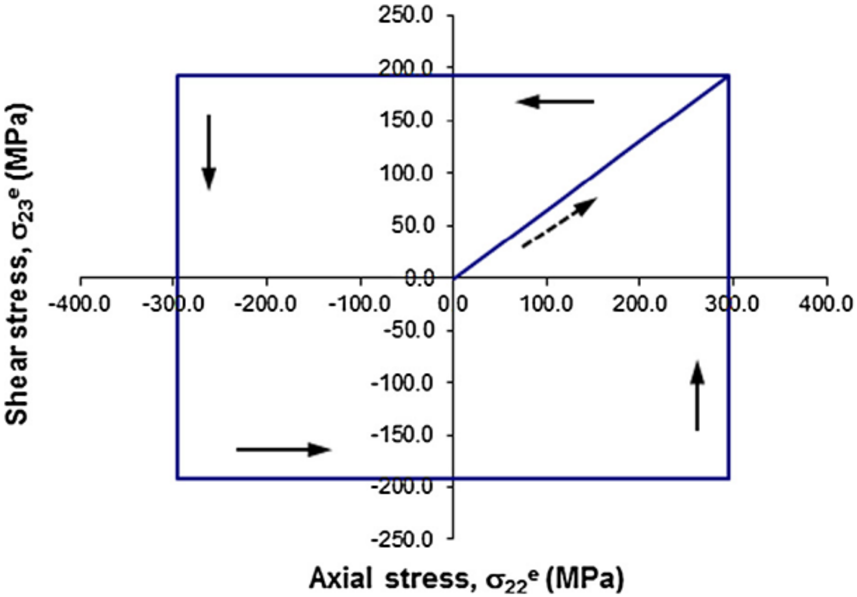


Fig. 9a. Box stress/load path – counter clockwise.

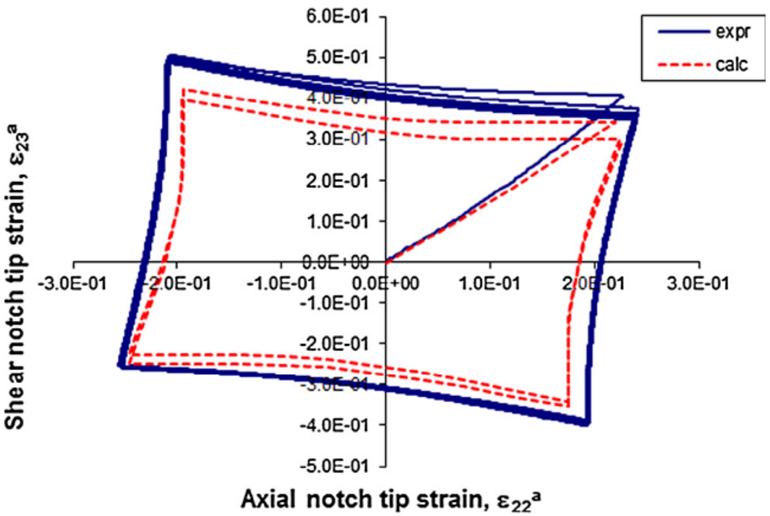
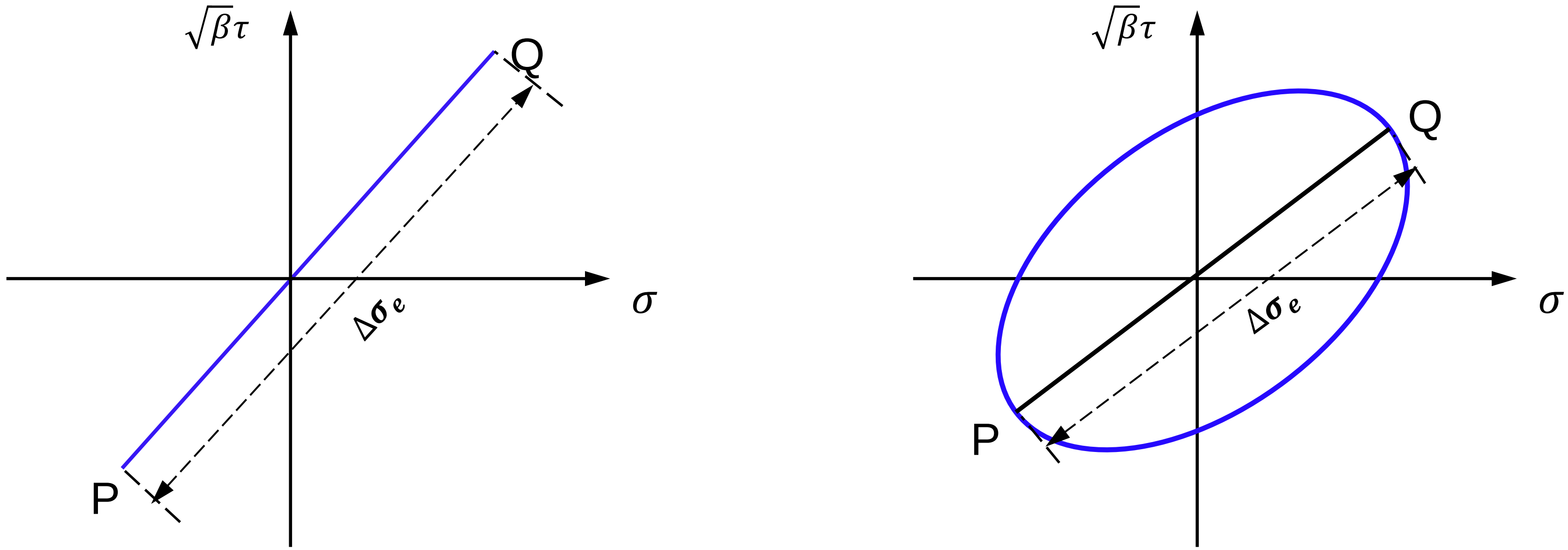


Fig. 9b. Experimental and calculated strain responses in the notch tip induced by the box input loading path – counter clockwise.

Background
---Load Paths for Proportional versus Non-Proportional loading



Proportional load path

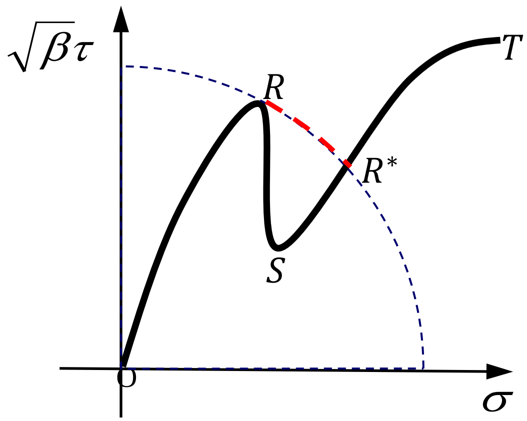
Non-proportional load path

Conventional effective stress range is still applicable, i. e.

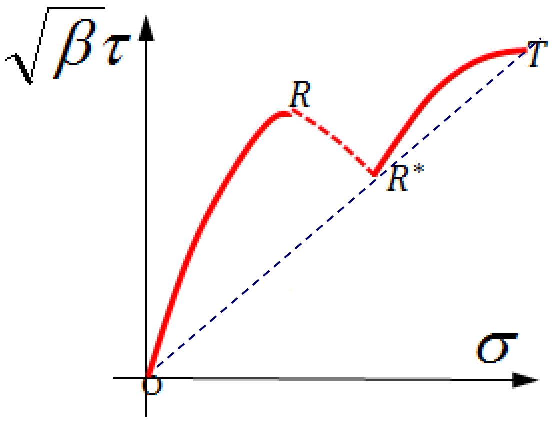
$$\Delta\sigma_e = \sqrt{\Delta\sigma^2 + \beta\Delta\tau^2}$$

$\beta = 3$ for von Mises stress range definition

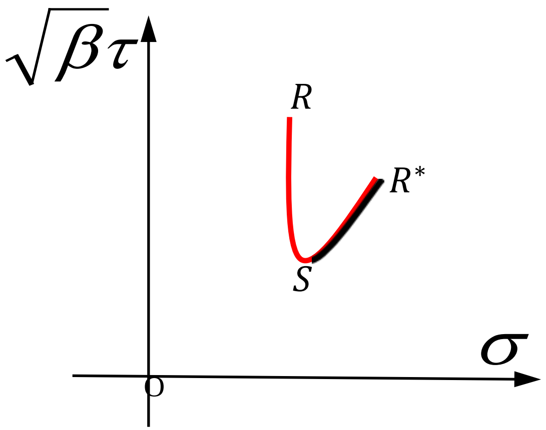
- How to define a stress range for Non-proportional load path?
- Can we still use $\Delta\sigma_e$ as a damage parameter?



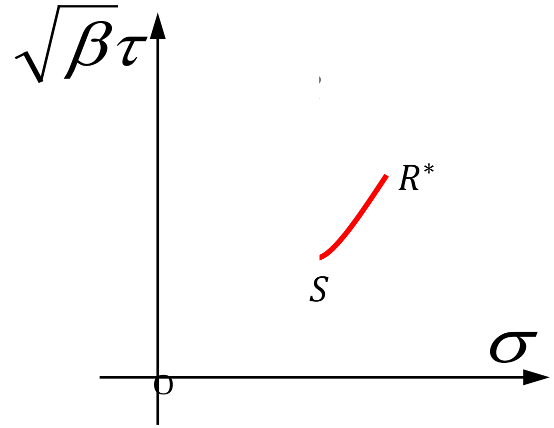
$\sqrt{\beta}\tau$
R
T
R^*
S
O
σ



$\sqrt{\beta}\tau$
R
T
R^*
σ
O



$\sqrt{\beta}\tau$
R
R^*
S
σ
O



$\sqrt{\beta}\tau$
R^*
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σ
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